THE APPLICATION OF THE NONLINEAR LEAST SQUARES FREQUENCY DOMAIN METHOD TO ESTIMATION OF THE MODAL MODEL PARAMETERS

Joanna Iwaniec, Tadeusz Uhl

*AGH University of Science and Technology, Department of Robotics an Machine Dynamics
al. Mickiewicza 30, 30-059 Cracow*

ABSTRACT

The least squares is the most popular algorithm for solving linear as well as nonlinear systems of equations. In the paper there are presented the theoretical basis of the Gauss – Newton and the Levenberg – Marquardt algorithms for solution estimation with the use of the nonlinear least squares method. The discussed schemes of proceeding usually allow the user to improve the quality of the estimated solutions. The analysis of frequency data measured on a real structure with the use of the created software realizing the Gauss – Newton algorithm in the Matlab environment is presented. The comparison of the analysis results estimated by the use of the proposed software and the LSCE algorithm implemented in the VIOMA toolbox is also included.

Keywords: modal analysis, nonlinear least squares frequency domain method, Gauss – Newton algorithm, model estimation.

List of most important symbols and abbreviations used in the paper:

$[]$, $\{\}$: matrix, vector,

$[]^T$, $\{\}^T$: transpose of a matrix, transpose of a vector,

$[]^*$, $\{\}^*$: conjugate matrix, conjugate vector,

$\{y\}$: vector of system response values,

$\{e\}$: vector of error values,

S: sum of square errors,

λ_r: system pole,

$\{\Psi_{ir}\}$: modal displacement vector (modal vector),

L_{rj}: modal participation factor,

UR_{ij}: upper residual term,

LR_{ij}: lower residual term,

SDT: Structural Dynamics Toolbox dedicated for Matlab,

SCSI: a junction between Siglab analyser and a superior unit (e. g. PC computer),

VIOMA (Virtual In – Operation Modal Analysis Toolbox): a set of Matlab functions designed for classical and in-operation modal analysis created in the Department of Robotics and Machine Dynamics of the AGH Technical University (Cracow).

1   Introduction

Dynamic properties of mechanical systems can stand for the basis of assessment of their technical state, degree of ware and exploitational parameters. Usually the analysis of dynamic properties is carried out for the model that represents mechanical structure dynamic behaviour. Nowadays it is a standard procedure to model mechanical systems with the use of modal models built on the basis of the finite element method or of the data measured on a real structure [Uhl T., 1997]. The process of structural modal creation is not only time-consuming but also the estimated model parameters should be fitted to data measured on a modelled structure. The model correctness can be verified by the comparison of frequency characteristics measured on a structure with model frequency characteristics obtained by synthesis.

The nonlinear least squares method, which is a subject of this paper, belongs to the group of methods that make it possible to identify modal model on the basis of data measured on a real structure.

2   Nonlinear least squares method

The least squares algorithm consists in estimation of the modal model parameters in such a way that a sum of square differences between a measured structure response and an estimated modal response is the smallest. The usage of square error values allows the analyst to emphasis differences between small and significant measurement errors and, in consequence, enables him to identify accidental errors and eliminate them from the further analysis [Ligęza J., 2002]. Moreover, all the computations are reduced to the simple transformations of the matrix algebra.

The least squares method can be used for solving systems of equations with the number of equations greater then the number of unknowns. In case of a system of linear equations, the method leads to the direct estimation of parameters. In case of a system of nonlinear equations it is necessary to assume the initial values; as a result of each iteration, new initial values, better 'fitted' to the measured characteristic are obtained.

The nonlinear least squares method is used in case when some parameters are nonlinear. The model is described by the equation [Norton J. P., 1986]:

(1) $\{y\} = f([U],\{\theta\}) + \{e\}$

where: {y}: vector of measured system response values, {f}: function modeling {y}, [U]: matrix containing known coefficients, {θ}: vector of unknown model coefficients, {e}: vector of measurement and modelling errors.

The obtained estimate $\{\hat{\theta}\}$ satisfies the so called generalised normal equation in the form of:

(2) $\quad \dfrac{\partial S}{\partial \{\theta\}} = -2[J_\theta f]^T (\{y\} - f([U], \{\theta\})) = 0$

where: $S = \{e\}^T \cdot \{e\}$: method error, $[J_\theta f]$: Jacobian matrix of the function f with respect to $\{\theta\}$; consecutive derivatives $\dfrac{\partial f_i}{\partial \theta_j}$ of the function f with respect to the unknown model coefficients $\{\theta\}$ form consecutive elements (i, j) of the Jacobian matrix.

Estimation of nonlinear model parameters is additionally complicated by the fact that Jacobian [J₀f] is a function of {θ}; as a result the generalised normal equation is nonlinear and requires iterative solving. Nonlinear equation coefficients are estimated for previously estimated linear coefficients.

2.1   Gauss – Newton algorithm

Nonlinear least squares solving with the use of the Gauss – Newton algorithm consists in gradual updating the vector of unknown model coefficients $\{\hat{\theta}\}$ in order to meet the condition $g(\{\hat{\theta}\}) = 0$. In the kth iteration, a vector of coefficients $\{\hat{\theta}^{(k-1)}\}$ is substituted by the vector $\{\hat{\theta}^{(k)}\}$ such that:

(3) $\quad \left\{\hat{\theta}^{(k)}\right\} = \left\{\hat{\theta}^{(k-1)}\right\} - \left[J_\theta g\big|_{\{\theta\}=\{\theta^{(k-1)}\}}\right]^{-1} \cdot g\left(\left\{\hat{\theta}^{(k-1)}\right\}\right)$

where: g({θ}): function dependent on {θ}, [J_θg]: Jacobian matrix of the function g with respect to {θ}.

On the basis of the normal equation (2) $g_i(\{\theta\})$:

(4) $\quad g_i(\{\theta\}) = -2 \cdot [J_\theta f]^T_{i,\ j=1,\ldots,n} \cdot (\{y\} - f([U], \{\theta\})),\quad n: number\ of\ [J_\theta f]\ matrix\ columns$

Element of the Jacobian matrix is described by the equation:

(5) $\quad \dfrac{\partial g_i}{\partial \theta_j} = 2 \sum_{l=1}^{N} \left\{ \dfrac{\partial f_l}{\partial \theta_i} \cdot \dfrac{\partial f_l}{\partial \theta_j} - \dfrac{\partial^2 f_l}{\partial \theta_i \partial \theta_j} \cdot (\{y_l\} - f_l([U], \{\theta\})) \right\}$

3

If the current estimate $\{\hat{\theta}^{(k-1)}\}$ gives relatively small error $\{y_l\} - f_l([U], \{\theta\})$ then in the equation (5) all the expressions containing second derivatives are neglected and the Jacobian is written in the following form:

$$(6) \qquad [J_\theta g] \approx 2\sum_{l=1}^{N} \frac{\partial f_l}{\partial \theta} \cdot \frac{\partial f_l^T}{\partial \theta} = 2[J_\theta f]^T [J_\theta f]$$

Having assumed notation $[J_\theta f] = [J^{(k-1)}]$, the equation describing the Gauss –Newton method has the form of:

$$(7) \qquad \left\{\hat{\theta}^{(k)}\right\} = \left\{\hat{\theta}^{(k-1)}\right\} + \left([J^{(k-1)}]^T [J^{(k-1)}]\right)^{-1} [J^{(k-1)}]^T \cdot \left(\{y\} - f^{(k-1)}([U], \{\theta\})\right)$$

where: $\{y\} - f^{(k-1)}([U], \{\theta\})$: method error in the $(k - 1)^{th}$ iteration.

In the kth iteration the unknown system parameters are estimated on the basis of the values of this parameters, Jacobian and the method error from the previous, $(k-1)^{th}$, iteration. If the sum of square errors increases in the consecutive iterations then the method is not convergent, which can result from the incorrect choice of poles initial values.

2.2 Levenberg – Marquardt algorithm

Taking small steps towards decreasing gradient enables the analyst to speed up reduction of the error value S in consecutive iterations from the value $\{\hat{\theta}^{(k-1)}\}$ to:

$$(8) \qquad \left\{\hat{\theta}^{(k)}\right\} = \left\{\hat{\theta}^{(k-1)}\right\} - \alpha \cdot \left(\frac{\partial S}{\partial \{\theta\}}\right)\Bigg|_{\{\theta\}=\left\{\hat{\theta}^{(k-1)}\right\}}$$

where: α: a small number that makes it possible to neglect all the expressions containing the second (or higher) derivatives. Unfortunately, in order to determine maximal possible value of α, it is necessary to investigate a local shape of the characteristic $S(\{\theta\})$. If a conservative value of α is chosen the progress is slow, which results from the fact that near the optimum gradient values are small.

The idea of the Levenberg – Marquardt algorithm is to find a compromise between direction of decreasing gradient and the direction given by the Gauss – Newton algorithm, which means that the following condition should be met:

$$(9) \qquad \left([J^{(k-1)}]^T \cdot [J^{(k-1)}] + \mu^{(k-1)} \cdot I\right) \cdot d_k = -\left(\frac{\partial S}{\partial \{\theta\}}\right)\Bigg|_{\{\theta\}=\left\{\hat{\theta}^{(k-1)}\right\}}$$

where: $d_k = \left\{ \overset{\wedge}{\theta}{}^{(k)} \right\} - \left\{ \overset{\wedge}{\theta}{}^{(k-1)} \right\}$: a step, $\mu^{(k-1)}$: positive value that controls both the magnitude and direction of d_k.

In case of choice of a small value $\mu^{(k-1)}$ a step is similar to the step from the Gauss – Newton algorithm. If the chosen value $\mu^{(k-1)}$ tends to infinity, the step d_k tends toward a vector of zeros and a steepest descent direction.

The value of μ is updated in each step depending on a progress from the previous step. Implementations of the Levenberg – Marquardt algorithm differ from each other in methods of μ updating (e.g. [MathWorks Inc., 2003]) – the maximal progress should take place in minimal number of iterations and the software protections in case of appearance of very small values of μ should be written.

3   Nonlinear least squares frequency domain method

The nonlinear least squares frequency domain method belongs to the group of global methods for multiple degree of freedom systems. It consists in formulating modal model equations and fitting, with the use of the nonlinear least squares method, this model parameters to data measured on the real system. The algorithm consists of a few basic steps:

–   choice of a frequency band in which the modal model is going to be identified,
–   choice of poles initial values,
–   model building (formulation of modal model equations),
–   model fitting to the measured data (with the use of the nonlinear least squares method),
–   defining the error between measured and estimated frequency characteristics,
–   minimization of the sum of square errors,
–   iteration repeating as long as the error is bigger than the assumed value.

For a given frequency band, a frequency response function (FRF) of a single degree of freedom system, defined between the point of response measurement i and the point in which the excitation is applied j, is approximated by the equation [Heylen W., et al., 1994]:

$$(10) \quad \left\{ H_{ij}^{E}(j\omega) \right\} = \sum_{r=1}^{Nm} \left(\frac{\{\Psi_{ir}\} L_{rj}}{(j\omega - \lambda_r)} + \frac{\{\Psi_{ir}^*\} L_{rj}^*}{(j\omega - \lambda_r^*)} \right) + UR_{ij} + \frac{LR_{ij}}{\omega^2}$$

where: $\left\{ H_{ij}^{E}(j\omega) \right\}$: measured system frequency response function; N_m: model order; $\{\Psi_{ir}\}$: modal vector; L_{rj}: modal participation factor; λ_r: system pole; UR_{ij}, LR_{ij}: upper and lower

residual terms approximating the contribution of modes lying above and below the measurement frequency band respectively.

The above equation can also be written in a simplified form:

$$(11) \quad \{H_{ij}^E(j\omega)\} = \sum_{r=1}^{Nm}\left(\frac{R_{ijr}}{(j\omega - \lambda_r)} + \frac{R_{ijr}^*}{(j\omega - \lambda_r^*)}\right) + UR_{ij} + \frac{LR_{ij}}{\omega^2}$$

where: R_{ijr} – modal residue.

The right side of the equation (11) is a function of frequency and unknown modal parameters of the system of interest:

$$(12) \quad \{G_{ij}(j\omega)\} = \{H_{ij}^E(j\omega, \lambda_r, \Psi_{ir}, L_{rj}, UR_{ij}, LR_{ij})\}, \quad r = 1, \ldots, N_m$$

or:

$$(13) \quad \{G_{ij}(j\omega)\} = \{H_{ij}^E(j\omega, \lambda_r, R_{ijr}, UR_{ij}, LR_{ij})\}, \quad r = 1, \ldots, N_m$$

The overall method square error for a given frequency band is given by the equation:

$$(14) \quad S_{ij} = \sum_{f=0}^{N_f} \{e_{ij}(j\cdot\omega_f)\}\cdot\{e_{ij}^*(j\cdot\omega_f)\}, \quad \{e_{ij}(j\cdot\omega_f)\} = \{H_{ij}^E(j\cdot\omega_f)\} - \{G_{ij}(j\cdot\omega_f)\}$$

where: $\langle\omega_0, \ldots, \omega_f\rangle$: given frequency band.

Unknown system parameters are estimated as a solution of the following system of equations:

$$(15) \quad \begin{cases} \dfrac{\partial S}{\partial \theta_1} = 0 \\ \quad\vdots \\ \dfrac{\partial S}{\partial \theta_n} = 0 \end{cases}$$

where: $\{\theta\} = [\theta_1, \ldots, \theta_n]$: vector of unknown model parameters.

In this paper the iterative Gauss – Newton algorithm is used for solving systems of nonlinear equations. This algorithm requires the choice of the unknown system poles initial values. Since the user guess of poles initial values is usually too distant from real values and results in method divergence, the LSFD method as a practical direct method of model identification has turned out to be troublesome and rather ineffective. The poles initial values (initial model) should be estimated by the use of either single degree of freedom methods (such as a circle fitting [Heylen W., et al., 1994], [Kurowski P., et al., 2001]) or multiple degree of freedom methods [Balmès E., 1996]. In such a case, it is worth to take note of the fact that stabilizing diagrams determined by the considered method are much more easy to interpret than those determined by classical modal analysis methods such as LSCE or ERA. Estimated pole lines are straight, do not 'branch off', which simplifies the choice of system

poles which is nowadays a critical stage of modal analysis. The application of the LSFD method with the use of the Gauss – Newton algorithm to the modal model identification proposed by the authors is depicted in the Fig.1.
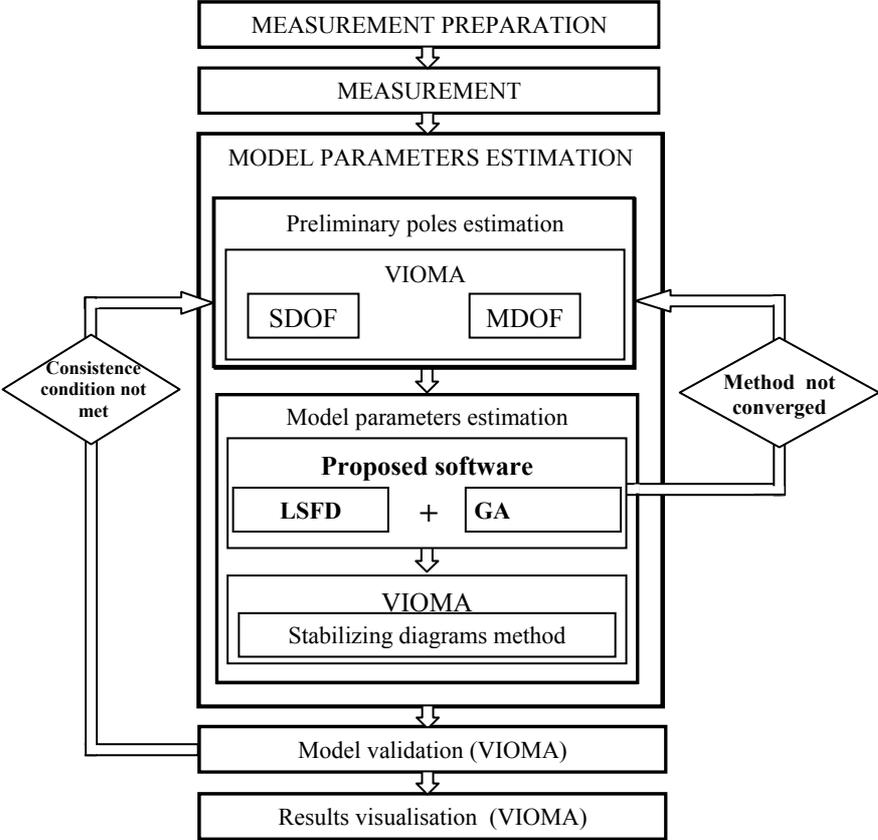
Fig. 1. The proposed application of the LSFD method with the use of the Gauss – Newton algorithm to the modal model identification

According to the Fig. 1, the proposed software realising the LSFD method with the use of the Gauss – Newton algorithm in Matlab is used for making the choice of correct poles values from the stabilizing diagrams more obvious for the analyst, which, in turn, leads to the quality improvement of the obtained modal models. In the kth method iteration, for poles initial values estimated by the use of methods implemented in the Vioma toolbox, values of model linear parameters are calculated. For this values the nonlinear parameters – system poles - are optimized in such a way that the model frequency response function is better fitted to the measured frequency response function than in the previous, $(k-1)^{th}$, iteration ($S^{(k-1)} > S^{(k)}$). The poles values estimated in the kth iteration are assumed as poles initial values in the $(k+1)^{th}$ iteration. Iterations are repeated as long as the error is bigger than the assumed value. The model is validated by the comparison of frequency characteristics measured on a structure with model frequency characteristics obtained by synthesis. If the consistence

condition is not met the preliminary poles estimation has to be repeated. Results visualisation stands for the last step of the analysis.

3.1 Accuracy of the nonlinear least squares frequency domain

The LSFD method is an 'interactive' method that requires careful analysis of the obtained solutions and, in case of appearance of significant approximation errors or lack of convergence, correction of problem formulation. Below there are presented schemes of proceeding, which usually enable the analyst to improve the quality of estimated solutions. To illustrate the theoretical considerations the characteristics of example data from the SDT user guide [Balmès E**.,** 1997] are quoted.

Measured amplitudes of frequency characteristics corresponding to the low frequency band are usually burden with errors resulting form signal processing or low sensitivity of piezoelectric sensors in this band.
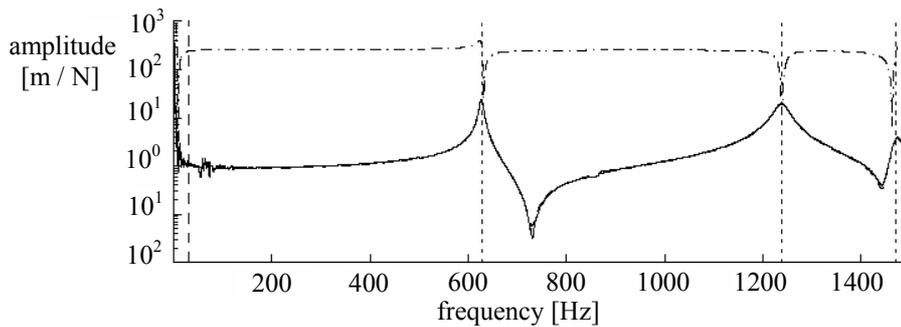
Fig. 2. The influence of taking into account a low frequency band on the LSFD accuracy. The results of analysis: ·−·−: including a low frequency band, −−−: excluding a low frequency band, —: measured characteristic.

In the Fig. 2. there are shown characteristics estimated by the use of the LSFD method including a low frequency band (·−·− line) and excluding this band (−−− line). In case of excluding a low frequency band data from the analysis, the estimated characteristic is fitted to the measured data accurately. Including a low frequency band data results in significant estimation errors.

The LSFD method is used for analysis in a chosen frequency band. Correct results are obtained only in case of introducing upper (UR_{ij}) and lower (LR_{ij}) residual terms that approximate the contribution of modes lying above and below the measurement frequency band.
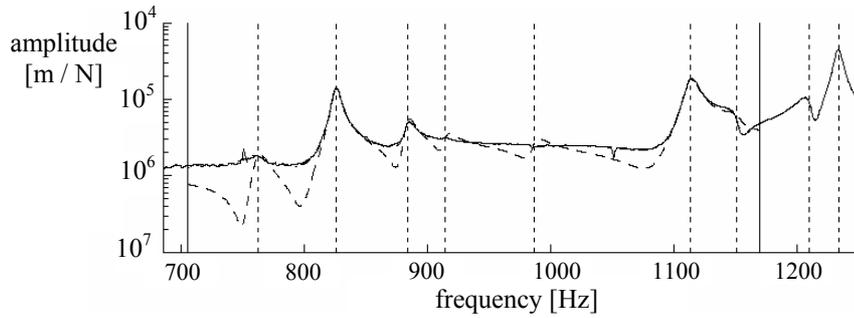
Fig. 3. The influence of introducing UR and LR coefficients on the LSFD accuracy. The results of analysis: ⋯⋯: including UR and LR coefficients, − − −: excluding UR and LR coefficients, —: measured characteristic.

The Fig. 3 illustrates the influence of introducing (or not) UR_{ij} and LR_{ij} coefficients on the estimated solution quality. While taking into account this coefficients (⋯⋯ characteristics), the estimated characteristic is fitted accurately to the measured data. Otherwise, the estimated solution (− − − characteristic) differs from the correct one.

Frequently, excluding the first resonance from the analyzed frequency band improves the quality of estimated solutions (Fig. 4).
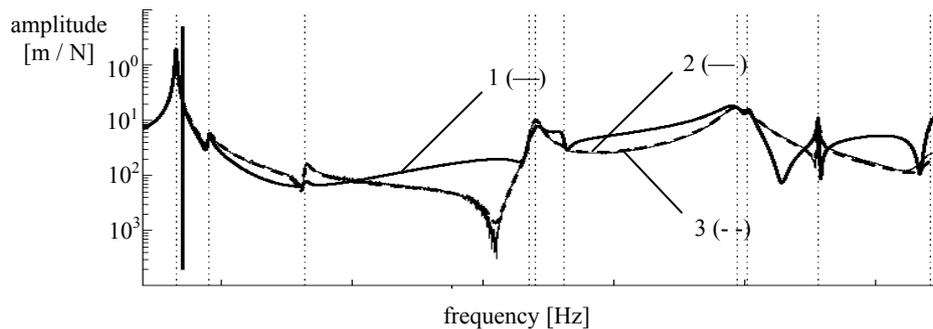
Fig. 4. The influence of taking into account the first resonance in the given frequency band on the LSFD accuracy. Results of analysis:1(—): with the first 'pick', 2 (- -): without the first 'pick', 3 (—): measured characteristic.

Such a phenomenon appears in case of characteristic burden with numerical errors of data processing at the band boundary.

3.2 The application of the LSFD method to estimation of the modal model of the real mechanical system

The nonlinear least squares frequency domain method LSFD with the use of the Gauss – Newton algorithm was implemented in the Matlab [Mrozek B., Mrozek Z., 1998] computational environment. In order to verify the created software, the analysis of a real structure – steel frame (Fig. 4) - was carried out. In the paper the results obtained for three points of the structure of interest are presented. Frequency response functions of the estimated

models, obtained by means of synthesis, and frequency response functions measured on the real structure are shown in the Fig. 6, Fig. 8, Fig. 10. The comparison of poles estimated by the use of the proposed software and the classical LSCE (Least Squares Complex Exponential) method implemented in the VIOMA toolbox [Uhl T., et al., 2000] is presented in the Table 1, Table 2 and Table 3. There are also shown the parameters of modal models obtained after 40 iterations of the proposed software.

The structure of interest was excited by the impulse force applied to the point 6 of the right bearing (Fig. 5, 'component right') in direction opposite to the x axis of the assumed coordinate system.
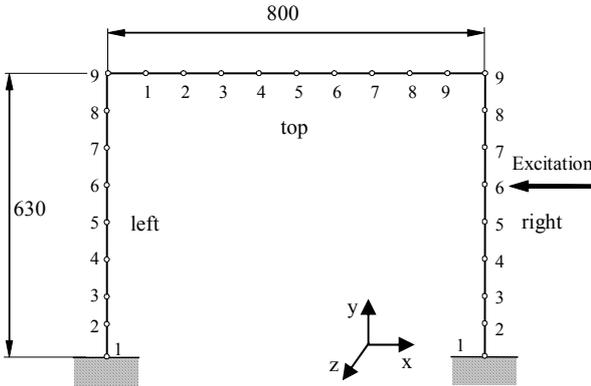
Fig. 5. Scheme of the structure of interest

A scheme of the test stand is shown in the Fig. 6 [Kurowski P., 2001].
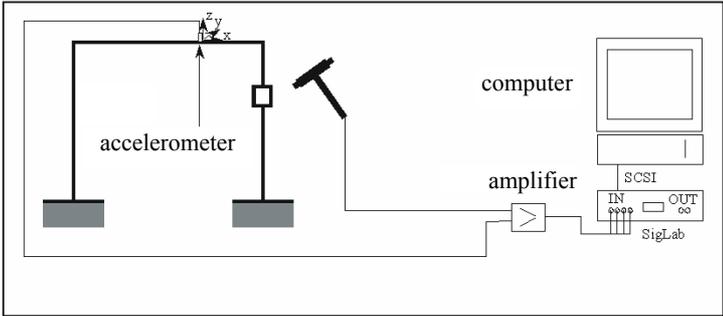
Fig. 6. Scheme of the test stand

The SigLab measurement analyzer (DSP Technology, model 2042) was used for registration of vibration accelerations in consecutive nodes as well as for estimation of spectral transfer functions necessary to perform modal analysis. System responses were measured with the use of piezoelectric accelerometers (PCB Piezotronics, USA) in each point in three directions.

For the frequency response function measured in the Y direction at the point 9 of the left bearing (left:9:Y), by the use of the LSFD method, the modal model was estimated. The model frequency response function is presented in the Fig. 7.
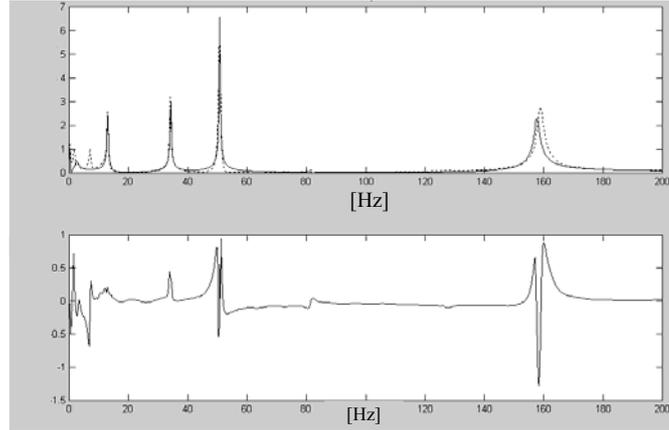
Fig. 7. Frequency response functions: ——: estimated by the use of the LSFD method, - - -:measured in the Y direction at the point 9 of the left bearing (upper diagram) and the relative error (lower diagram).

Poles estimated by the use of the LSFD and classical LSCE methods are presented in the Table 1. There are also shown the parameters of the model fitted to the measured characteristic by the use of the LSFD method.

left:9:Y	LSCE		LSFD			
	f [Hz]	ζ [%]	f [Hz]	ζ [%]	parameter	value
pole I	12,884	1,12	12,884	0,86	R_1	- 0,663 - 2,431i
pole II	34,884	0,39	34,884	0,39	R_2	0,596 + 3,016i
pole III	50,769	0,36	50,769	0,24	R_3	3,842 - 3,421i
pole IV	81,690	0,92	81,690	0,52	R_4	1,567 - 0,095i
pole V	158,982	0,64	158,982	0,54	R_5	- 0,136 - 0,090i
sum of square errors: S = 58,15						
upper residual term UR: - 0,053 + 0,011i						
lower residual term LR: - 2,750 + 1,095i						

Table 1. Poles estimated by the use of the LSFD and LSCE method. Parameters of the model fitted by the use of the LSFD method to the frequency response function measured at the left:9:Y point.

In the Fig. 8 there are presented the stabilizing diagrams determined by the use of the classical LSCE and LSFD methods.

11
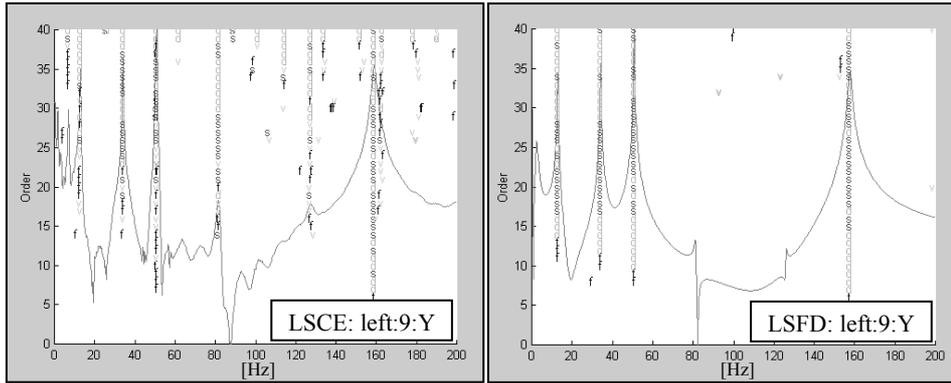
Fig. 8. Stabilizing diagrams estimated by the use of the LSCE and LSFD methods for the frequency response function measured at the point left:9:Y.

In the Fig. 9 there is presented a frequency response function of a modal model estimated by the use of the LSFD method for the data measured in the X direction at the point 4 of the steel frame horizontal beam (point top:4:X).
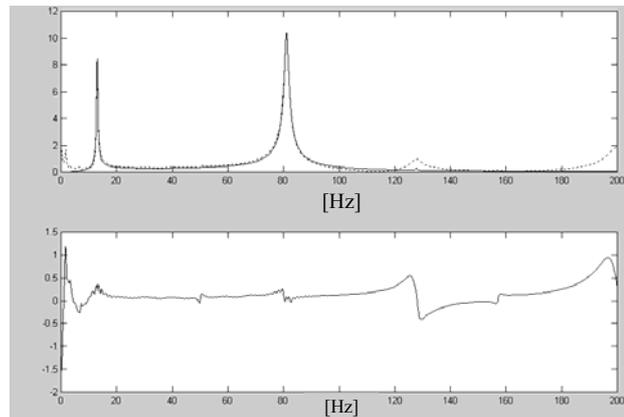
Fig. 9. Frequency response functions: ——: estimated by the use of the LSFD method, - - -: measured at the point top:4:X (upper diagram) and the relative error(lower diagram).

Values of poles estimated by the use of the LSFD and LSCE methods as well as the other parameters of the model obtained after 40 iterations of the LSFD method for the considered characteristic are gathered in the Table 2.

top:4:X	LSCE		LSFD			
	f [Hz]	ζ [%]	f [Hz]	ζ [%]	parameter	value
pole I	12,879	0,83	12,879	0,83	R_1	-0,274 + 0,066i
pole II	50,188	0,62	-	-	-	-
pole III	81,133	0,98	81,133	0,95	R_2	2,153 – 50,652i
pole IV	127,686	1,75	127,686	0,60	R_3	- 1,263 + 8,363i
pole V	157,101	0,58	-	-	-	-
sum of square errors: S = 113,90						
upper residual term UR: - 0,138 + 0,088i						
lower residual term LR: - 4,290 – 1,203i						

Table 2. Poles estimated by the use of the LSFD and LSCE method. Parameters of the model fitted by the use of the LSFD method to the frequency response function measured at the top:4:X point.

In the Fig. 10 there are presented the stabilizing diagrams obtained by the use of the LSCE and LSFD methods for the data measured at the point top:4:X.
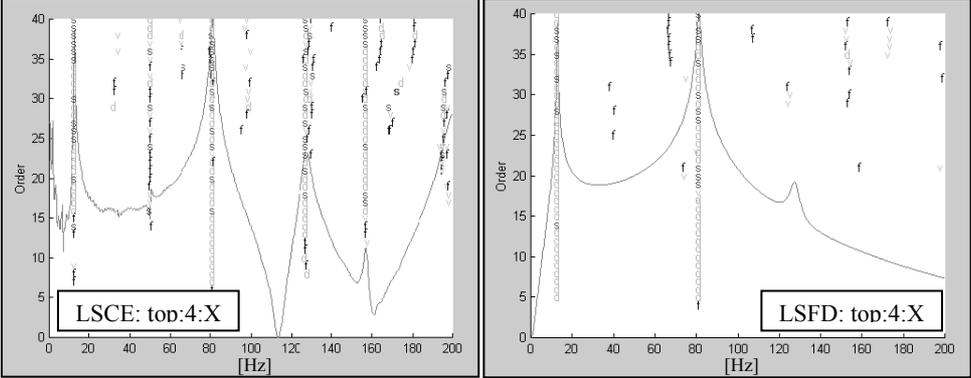
Fig. 10. Stabilizing diagrams estimated by the use of the LSCE and LSFD methods for the data measured at the point top:4:X.

For a frequency response function measured in the –Z direction at the point 4 of the steel frame right bearing (point right:4:-Z), by the use of the LSFD method, a modal model of a frequency response function shown in the Fig. 11 was obtained.
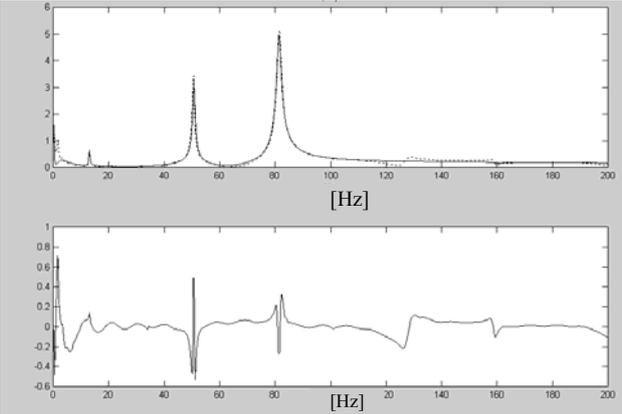
Fig. 11. Frequency response functions: —— : estimated by the use of the LSFD method, - - -: measured at the point right:4: –Z of the steel frame (upper diagram) and the relative error (lower diagram).

In the Table 3 there are gathered the values of poles estimated for a considered characteristic by the use of the LSFD and LSCE methods as well as the parameters of the model frequency response function estimated by the use of the LSFD method.

right:4:-Z	LSCE		LSFD			
	f [Hz]	ζ [%]	f [Hz]	ζ [%]	parameter	value
pole I	12,889	1,28	12,889	0,91	R_1	0,470 + 5,212i
pole II	50,579	0,60	50,579	0,69	R_2	3,129 + 25,119i
pole III	-	-	81,388	0,98	R_3	- 0,241 - 0,671i
pole IV	127,579	1,64	-	-	-	-
sum of square errors: S = 14,83						
upper residual term UR: - 0,144 + 0,027i						
lower residual term LR: - 3,586 + 1,074i						

Table 3. Poles estimated by the use of the LSFD and LSCE method. Parameters of the model fitted by the use of the LSFD method to the frequency response function measured at the point right:4:-Z.

In the Fig. 12 there are presented the stabilizing diagrams for the data measured at the point right:9:Y.
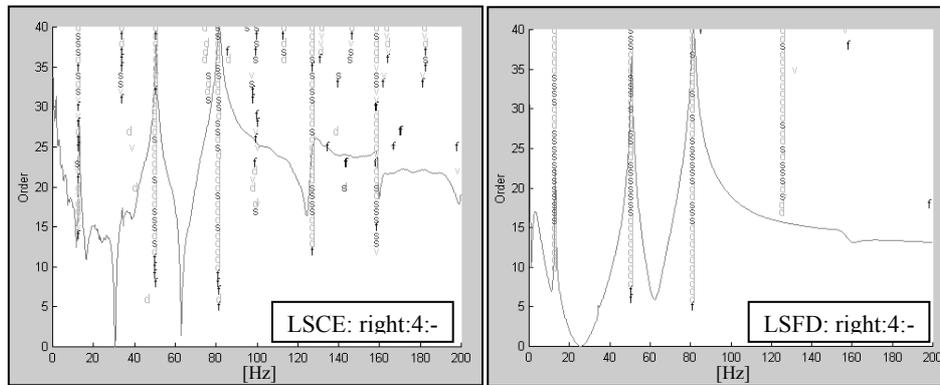
Fig. 22. Stabilizing diagrams estimated by the use of the LSCE and LSFD methods for the data measured at the point rigth:4:-Z.

The performed research revealed that the results obtained by the use of the LSFD method are burden with errors mainly for resonant areas lying in a high frequency band.

Stabilizing diagrams estimated by the use of the LSFD method are easier to interpret than those obtained by the use of the LSCE method: estimated pole lines are straight, not 'branched off', which simplifies the choice of system poles.

4   Conclusions and final remarks

In the paper there are presented the issues concerning the nonlinear least squares method as well as the Gauss – Newton and the Levenberg – Marquardt algorithms that provide numerical solution for this method. The authors proposed the application of the nonlinear least squares frequency domain method (LSFD) with the use of the Gauss – Newton algorithm to make the choice of correct poles values from the stabilizing diagrams more obvious for the analyst, which is considered as a critical stage of modern modal analysis.

Such a usage of the considered method can lead to the quality improvement of the obtained modal models.

The created software was verified for the frequency response functions measured on the steel frame. In order to shorten the estimation time and to assure the method convergence as the poles initial values the poles estimated preliminarily by the use of the classical LSCE method implemented in the VIOMA toolbox were assumed. Since all the poles are updated simultaneously, the number of necessary iterations rarely exceeds 40 to 50 iterations. For the considered structure the problem of ill-conditioned matrixes did not appear.

References

Balmès E., 1997, *Structural Dynamics Toolbox for Use With Matlab, User's Guide, Version 3*, Scientific Software Group, Sèvres, France.
Balmès E., 1996, Frequency domain identification of structural dynamics using the pole / residue parametrization, *International Modal Analysis Conference*, Dearborn.
Giergiel J., Uhl T., 1990, *Identification of mechanical systems* (in Polish), Państwowe Wydawnictwo Naukowe, Warsaw.
Heylen W., Lammens S., Sas P., 1994, *Modal Analysis Theory and Testing*, KUL Leuwen.
Kurowski P., 2001, *Identification of modal models of mechanical structures on the basis of the in-operational measurements* (AGH Ph. thesis in Polish), Cracow.
Kurowski P., Ligęza J., Uhl T., 2001, The usage of the Matlab environment for modal model parameters estimation for single degree of freedom systems (in Polish), *5th Scientific Conference: Experimental methods in machine design and exploitation*, Wrocław – Szklarska Poręba, 65 – 77.
Ligęza J., 2002, *Implementation of modal analysis algorithms in Matlab* (AGH master's thesis in Polish), Cracow.
MathWorks Inc., 2003, *Optimization Toolbox User's Guide*, http://www.mathworks.com /access/helpdesk/help/toolbox/optim/optim.shtml
Mrozek B., Mrozek Z., 1998, Matlab 5.x Simulink 2.x user guide (in Polish), Wydawnictwo PLJ, Warsaw.
Norton J. P., 1992, *An Introduction to Identyfication*, AP (Academic Press), Londyn.
Uhl T., **Lisowski W., Kurowski P.,** 2000, In–operation modal analysis and its applications, Katedra Robotyki i Dynamiki Maszyn, AGH, Cracow.
Uhl T., 1997, *Computer aided identification of mechanical structure models (in Polish)*, WNT, Warsaw.